

# Where to Get a Charged EV Battery: A Route to Follow as if It Were Your Own Advice

Francesco Malandrino, Claudio Casetti, Carla-Fabiana Chiasserini, Massimo Reinieri  
Politecnico di Torino, Italy

**Abstract**—We address the problem of Electric Vehicle (EV) drivers' assistance through ITS. Drivers of EVs that are low in battery may ask a navigation service for advice on which charging station to use and which route to take. A rational driver will follow the advice if there is no alternative choice that lets her reach its destination in a shorter time, i.e., in game-theory terms, if such advice corresponds to a Nash-equilibrium strategy. Thus, we envision two game-theoretic models, namely, a congestion game and a game with congestion-averse utilities, both known to admit at least one pure-strategy Nash equilibrium. The former represents a practical scenario with a high level of realism, although at a high computational price. The latter neglects some features of the real-world scenario but it exhibits very low complexity, and is shown to be an excellent approximation of the former. Importantly, we show that the average per-EV trip time yielded by the Nash equilibria is very close to the one attained by solving a centralized optimization problem that minimizes such a quantity.

## I. INTRODUCTION

It is conceivable that in ten years' time Electric Vehicles (EVs) will take over the streets. Old-fashioned gas pumps will be gradually phased out by public charging stations, with electric outlets popping up in places such as curbside parking, parking lots and cab stands. Still, worries about vehicle range and availability of charging stations may persist and drivers will be forced to commute around such availability, at least early on in charging station development. Finally, it is not clear when the "time consuming" tag will be removed from the task of car recharging.

Traditional navigation services could be integrated by ICT and ITS (Intelligent Transportation Systems) with the information provided by roadside network infrastructure and on-board user terminals through wireless communication [1], [2]. A Central Controller (CC) could collect information on the vehicular traffic conditions and on the occupancy status of the charging stations through ITS facilities. Then, EV drivers that need to recharge their batteries could send a request to the CC and ask for advice on the specific charging station to choose and the route to take.

The key point in this scenario, however, is that drivers that resort to such a navigation service will very likely behave as self-interested users, who aim at reaching their destination in the shortest possible time. Thus, they will follow the CC's advice only if they find it convenient to themselves.

This is an aspect that so far has been scarcely considered in the literature. Indeed, existing works leave the burden of selecting the charging station to the drivers and do not account for the trip time associated to different alternatives [3], [4], or they focus on the EV consumption and its impact on

the power grid but neglect the time the EVs may have to wait in line at the charging station or the fact that EVs may act strategically [5], [6].

In this work, we aim at filling this gap. The advice provided by the CC may not conform to the interests of EV drivers when it is obtained by solving a centralized optimization problem that, e.g., minimizes the average per-EV trip time or the maximum EV expected trip time. We demonstrate instead that the above requirement is satisfied when the problem is modeled as a non-cooperative game. Specifically, we resort to a congestion game [7] and a game with congestion-averse utilities [8], where the players are the EVs that need to recharge their batteries. In such games, the decision to be made concerns the charging station that an EV should use, along with the route to take passing through such a station, so as to minimize the EV's trip time. The two game models exhibit a different level of realism and complexity; however, for both of them, we show that, when the CC uses the game solution to provide advice to the EVs, the following facts hold.

- (i) The navigation strategies suggested by the CC correspond to Nash Equilibrium (NE) strategy profiles<sup>1</sup>, i.e., each EV finds the suggestion by the CC convenient to itself and is willing to adhere to it.
- (ii) The advice provided by the CC leads to an average per-EV trip time that is very close to the minimum obtained by solving a centralized optimization problem, and much shorter than the one EV drivers can obtain by adopting a greedy approach (e.g., select the closest charging station). This is highly desirable since the shortening of the average per-EV trip time contributes to reducing road congestion and energy consumption due to EVs.

The remainder of the paper is organized as follows. The system scenario is introduced in Sec. II, along with the statement of the problem under study. The game-theoretic approach that we adopt can be found in Sec. III. In Sec. IV, we show the benefits of the proposed method in terms of per-EV trip time, using a realistic simulation scenario. We draw our conclusions and discuss future work in Sec. V.

## II. SYSTEM SCENARIO AND PROBLEM STATEMENT

We consider a road topology including a set of *road segments*  $\mathcal{L}$  and a set of *charging stations*  $\mathcal{C}$ . Any ordered sequence of adjacent segments  $l \in \mathcal{L}$  is said to form a *route*.

<sup>1</sup>An NE is a game solution, in which no player can gain anything by unilaterally changing his own strategy.

Among all vehicles that travel across the topology, we identify the following two categories: (i) non-EVs or EVs that are not interested in using a charging station; (ii) EVs with low battery that use the navigation service to select a charging station, and that, if they find it convenient, may deviate from their original route to reach a charging station.

For the vehicles that stop at a charging station, it is fair to assume that their battery is replaced with a fully-charged one. Only in the unlikely case where no one is available, is the EV battery recharged. This choice is due to the charging times approaching half an hour, according to today's fast recharge technology [9]. Charging stations may have a number of replacing stalls (hereinafter servers), possibly varying from one station to another. At a charging station, an EV will incur a waiting time that depends on the occupancy of the station, the service time and the number of available servers. We assume EV drivers to be that self-interested users, i.e., to aim at *minimizing the total trip time* toward their intended destination.

In the most general case, such EV drivers will ask the advice of the CC to make a decision on the charging station (among many) to use and which route to take, including their current position and final destination in the request. The CC provides advice leveraging its knowledge of the road topology and traffic conditions, as well as of the locations of the charging stations, their current occupancy and availability of fully-charged batteries. All rational, self-interested EVs will be willing to follow such suggestions if they conform to their own interest.

A natural choice to solve the problem of selecting the charging station for each EV, and the corresponding route, would be to let the CC formulate an optimization problem that minimizes the average per-EV trip time. It is, however, easy to show that in general such an approach yields solutions that EV drivers may find not convenient to themselves, hence to which they will not adhere. The same observation holds in the case where the CC tries to minimize the maximum EV expected trip time.

The approach we propose is different. We model the problem of selecting the charging station, and the corresponding route, as a non-cooperative game, in which the players are the EV drivers that resort to the navigation system for advice. Then, we look for a strategy profile that is an NE and is convenient from the viewpoint of the system performance, and we take this as a solution to the problem. Being an NE, self-interested drivers will adhere to it. It is clear, however, that a game-theoretic approach does not ensure that the average per-EV trip time is minimized. Nevertheless, in Sec. IV we show that, even in real-world scenarios, the average per-EV trip time obtained through our game-theoretic approach is remarkably close to the optimum.

Finally, though the game could be solved by the EVs themselves if armed with the required information, we assume it is the CC that collects all the information, processes it and solves the game so as to provide the EV drivers with the strategy to adopt. This implies that the proposed

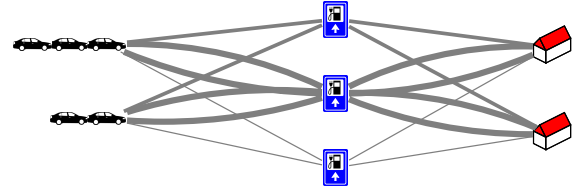


Fig. 1. Abstract representation of the scenario where each EV may take different routes to a given charging station and from there to its destination.

mechanism neither significantly increases the system overhead (except for providing the strategy to EV drivers) nor requires EV drivers to exchange sensitive information about themselves, or make any computation to take a decision.

### III. THE RECHARGING GAME

Assume that the CC processes the requests received from EVs with low battery every  $T$  seconds. We denote the set of EVs that ask for advice during a  $T$ -second time period by  $\mathcal{N}$ , and its cardinality by  $N$ . Consider the most general case in which each of the  $N$  EVs may reach several charging stations and take different routes to a given station, as well from the station to its final destination. For clarity, we depict an abstract representation of such a scenario in Fig. 1.

In the figure, lines connecting vehicles with charging stations, and the latter with final destinations, represent the possible road segments that EVs can take to or from the charging station. Thicker lines correspond to more congested (hence slower) road segments. We then consider the  $N$  EVs to be the players of a congestion game [7], i.e., a non-cooperative game, in which players strategically choose from a set of facilities and derive utilities that depend (in an arbitrary way) on the congestion level of each facility, i.e., on the number of players using it. Congestion games are of particular interest to us since they have been proved [7] to admit at least one pure-strategy<sup>2</sup> NE.

#### A. The Congestion Game

A congestion game is defined by the 4-tuple

$$\Gamma = (\mathcal{N}, \mathcal{F}, \{\mathcal{S}_i\}, \{\tau_i(n_i), \eta_c(n_c)\}) , \quad (1)$$

whose elements in our case are as follows.

- (i) The set of players,  $\mathcal{N}$ , which, as mentioned, correspond to the EVs using the navigation service.
- (ii) The set of facilities,  $\mathcal{F}$ , which is composed of all possible charging stations and road segments included in the road topology, i.e.,  $\mathcal{F} = \mathcal{C} \cup \mathcal{L}$ . Given  $\mathcal{F}$ , for each player  $i \in \mathcal{N}$ , a subset  $\mathcal{F}_i \subseteq \mathcal{F}$  can be identified, including all facilities that EV  $i$  can reach and use on its way to the destination. Clearly, if the road topology is fully connected, then  $\mathcal{F}_i = \mathcal{F}$ ,  $\forall i \in \mathcal{N}$ .
- (iii) The set of viable strategies for EV  $i$ , i.e., all groups of facilities that can be used by  $i$ ,  $\mathcal{S}_i \subseteq \mathcal{P}(\mathcal{F}_i)$  (where  $\mathcal{P}(\mathcal{F}_i)$  is the set of all possible partitions of  $\mathcal{F}_i$ ).

<sup>2</sup>A pure-strategy NE is a deterministic solution, as opposed to a probabilistic one (e.g., go to charging station  $c_x$ , rather than go to  $c_x$  with probability 0.5).

In our context, each strategy in  $\mathcal{S}_i$  is composed of: (a) one of the charging stations that EV  $i$  can reach, along with (b) the road segments forming a route that allows  $i$  to go from its current position to the selected charging station, and from there to its final destination.

For each strategy, the associated utility is the sum of the utilities of each selected facility (either a charging station or a road segment). The utility of a facility is its negated cost. Such a cost is defined as a function mapping the number  $n_f$  of players selecting the facility onto a time delay in  $\mathbb{R}$ . Note that the cost of a facility does not depend on the player identity, but only on the number of vehicles using the facility. The cost of a strategy is thus the sum of 1) the expected waiting time and the service time at the corresponding charging station, and of 2) the travel time on the associated route, from current road segment to destination, via the charging station. We denote the former by  $\eta_c(n_c)$ , with  $c \in \mathcal{C}$  and  $n_c$  being the number of players selecting station  $c$ . We denote the latter by  $\sum_l \tau_l(n_l)$ , with the  $l$ 's being the road segments in the chosen route and  $n_l$  the number of players taking segment  $l$ .

Furthermore, in accordance with the scenario detailed in Sec. II, we write  $\eta_c(n_c)$  so as to account for (a) the number of servers at station  $c$ ,  $K_c$ , (b) the service time, (c) the number of fully-charged batteries currently available at  $c$ ,  $B_c$ , and (d) the waiting time before an EV can be served. Specifically, we write  $\eta_c(n_c)$  as:

$$\eta_c(n_c) = \begin{cases} \sigma & \text{if } w_c < K_c \\ \sigma + \frac{\sigma}{2} & \text{if } K_c \leq w_c < 2K_c \\ \sigma + \frac{\sigma}{2} + \left\lfloor \frac{w_c - K_c}{K_c} \right\rfloor \sigma & \text{if } 2K_c \leq w_c < B_c \\ \rho & \text{if } w_c \geq B_c \end{cases} \quad (2)$$

with  $w_c$  being the expected number of EVs that the generic player finds at the charging station upon its arrival. Such a value is given by:  $w_c = m_c + n_c/2$ , where  $m_c$  is the number of non-player EVs that the CC estimates to be already at the station upon the arrival of the generic player, and  $n_c/2$  is the expected number of other players that have already reached  $c$ , if  $n_c$  players decide to use such a station. Note that  $w_c$  does not account for the precise order of arrival of the single players since the cost cannot depend on the player identity.

In (2) the first line corresponds to the case where the generic player finds an idle server, hence its stopping time at  $c$  coincides with the time necessary for battery replacement,  $\sigma$ , which is assumed to be constant. The second line, instead, represents the case where all servers are busy but the player finds a server with nobody else waiting to be served (the expected remaining service time of the EV that is currently under service is  $\sigma/2$ ). The third line refers to all servers at  $c$  being busy, with EVs already waiting there to be served. Thus, assuming a balanced load, the expression includes the expected time that the generic player has to spend in line. Finally, the last line applies when no more fully-charged batteries are available at the station, and the generic player has to recharge its battery, in a time that is assumed to be

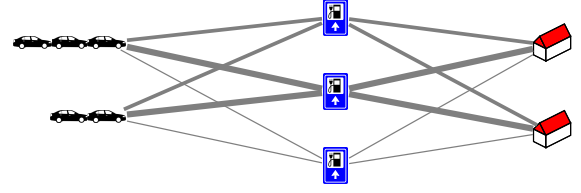


Fig. 2. Abstract representation of the scenario with only one possible route for each EV to a given charging station, and from there to its destination.

constant and equal to  $\rho$ .

### B. Game Model with Congestion-averse Utilities

Let us now consider the same scenario as above, but assume that, for every EV-charging station pair, there exists only one possible route to take, as depicted in Fig. 2. We stress that, although simpler, such a model is still realistic if the CC associates to the EV-charging station pair the route deemed to be the fastest one, according to the road information collected in the previous  $T$  seconds. As confirmed by our results derived in real-world scenarios (see Sec. IV), such a simplification is unlikely to significantly impair the performance.

Under the above assumption, the system can be modeled as a game with congestion-averse utilities (CAG), for which NEs are pure-strategies and can be found in polynomial time [8]. The game is defined as a 4-uple similar to  $\Gamma$ , as in (1), however, two main differences exist between CAGs and congestion games:

- (a) in CAGs, it must hold that  $\mathcal{S}_i = \mathcal{P}(\mathcal{F}_i)$ ,  $\forall i \in \mathcal{N}$ , i.e., all partitions of  $\mathcal{F}_i$  are possible strategies, and
- (b) the facilities cost can depend on player identities.

The first difference implies that, for each player  $i$ , the CC has to consider as viable strategies not a subset but all possible partitions of  $\mathcal{F}_i$ . A set  $\mathcal{F}$  defined as in the case of the congestion game would force the CC to consider non-meaningful strategies where an EV stops at more than one charging station, located either on the same route or on different routes. In order to overcome this issue, as a first step we redefine the set of facilities as  $\mathcal{F} = \mathcal{C}$ , i.e., we remove the road segments and consider only the charging stations. It follows that the set of facilities that the generic player  $i$  can use,  $\mathcal{F}_i$ , is now given by just the charging stations that the EV can reach. This is a viable choice since, per the initial assumption in this subsection, each EV-charging station pairs is implicitly, and univocally, associated to one route only. As a second step, we prove the lemma below.

**Lemma 1:** Consider the game with congestion-averse utilities introduced above, in which each facility has a cost greater than 0. Then, in order to identify a pure-strategy NE, for any player  $i \in \mathcal{N}$  it is sufficient to examine the subset of viable strategies  $\overline{\mathcal{S}}_i \subseteq \mathcal{S}_i$ , such that each strategy in  $\overline{\mathcal{S}}_i$  includes one facility only.

*Proof:* Please see [10]. ■

Based on the above result, we can limit our attention to the set of strategies  $\overline{\mathcal{S}}_i$ , which includes only partitions of  $\mathcal{F}_i$  with cardinality equal to 1, and each of them corresponding to only one of the charging stations that EV  $i$  can reach.

Next, we leverage the second difference between CAGs and congestion games, i.e., the fact that in CAGs utilities can depend on the player identity. In particular, we define the cost of a charging station  $c$ , which can be used by player  $i$ , as the total trip time  $i$  would incur, and we write it as:

$$\eta_{i,c}(n_c^{(i)}) + \tau_{i,c}. \quad (3)$$

In (3), the first term is the sum of the delay due to the expected waiting time and the charging time at station  $c$ , while the second term is the travel time through the route associated to the EV-charging station pair  $(i, c)$ . Note that both terms depend on the player identity  $i$ ; furthermore, the following remarks hold.

(a)  $\eta_{i,c}(n_c^{(i)})$  can be obtained from (2) by replacing  $w_c$  with  $m_c^{(i)} + n_c^{(i)}$ , and  $\rho$  with  $\rho_i$ . Indeed, now the CC can account for the number  $m_c^{(i)}$  of non-player EVs that it estimates to be at the station upon the arrival of player  $i$ . Similarly,  $n_c^{(i)}$  is the number of players that the CC estimates to arrive at  $c$  before player  $i$  does. Finally, the recharging time  $\rho_i$  may be different from one player to another, and depend on the remaining battery charge of the EV.

(b) The travel time  $\tau_{i,c}$ , associated to the EV-charging station pair  $(i, c)$ , does not depend on  $n_c^{(i)}$ , as it now accounts for the vehicular traffic intensity due to all non-player vehicles only (i.e., the contribution of the players is neglected). Indeed, the CAG model cannot track the contribution to the traffic intensity due to players selecting different charging stations but whose route partially overlap. The impact of such an approximation is very limited since typically the number of players is much smaller than the number of all other vehicles traveling over the road topology (see also the results in Sec. IV).

As mentioned, in the case of CAGs, pure-strategy NEs can be found in polynomial time [8], thus the CC can solve the game with low complexity. In the following, we show how good the solutions of such games are from the system performance viewpoint, and that, in spite of its low complexity, the CAG model approximates very well the previous (most general) scenario where multiple routes may exist for any EV-charging station pair.

#### IV. RESULTS

We evaluate our game-theoretic approach on a real-world road topology representing a  $3 \times 2 \text{ km}^2$  section of the urban area of Ingolstadt, Germany, depicted in Fig. 3. The vehicle mobility has been synthetically generated using the SUMO simulator, with a time granularity of 0.1 s. The mobility trace is representative of 30-minute-long road traffic and of average traffic intensity in the area. The use of synthetic trace over real-world ones allows us to tweak number of vehicles simultaneously present in our road topology and to reroute them when needed.

The scenario includes 6 charging stations on the main arteries of the road topology (red dots in Fig. 3). Two stations have 2 servers, other two have 6 servers and the remaining ones have 4 and 10 servers each. We assume that

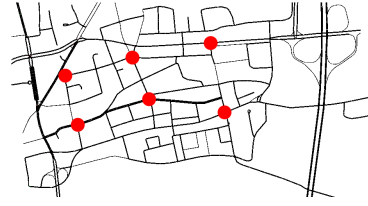


Fig. 3. Road topology: red dots highlight the six charging stations.

fully-charged batteries are always available at the charging stations, thus the service time is considered to be constant and equal to 3 minutes.

Without loss of generality, all vehicles are assumed to be electric. The average number of EVs that resort to the navigation service is a varying parameter in our simulations. The time instant at which an EV enters the low-battery status and asks the CC for advice is uniformly distributed over its trip time. Also we assume that all EV drivers are rational.

We consider that the CC receives information on the number of EVs currently waiting at a charging station to be served, as well as on the traffic conditions, every 10 seconds. The requests for the navigation service sent by the EVs are instead processed by the CC every  $T = 60 \text{ s}$ . All information is exchanged over the cellular network.

In order to derive the results, we proceed as follows. Every time interval  $T$ , the CC solves the game considering as players the EVs from which it has received a request. To do so, the CC starts from a random strategy profile, i.e., a random assignment of the facilities to the players. Player payoffs (i.e., trip times) are then computed through SUMO in the scenario described above. Given the current strategy profile and player payoffs, the CC examines other strategies according to the solution algorithm in [8] for the CAG, and to the one in [11, Ch.7] for the congestion game. For every strategy, player payoffs are computed via SUMO as before. If a more convenient strategy is found for any of the players, then the new strategy is adopted and the whole procedure is repeated until an NE is reached. Unless otherwise specified, we consider that the CC takes the first NE it finds as the solution of the game. For both the CAG and the congestion game, we calculate the per-player trip time associated to such a solution. All results are averaged over 10 runs. We compare such values with the trip time obtained through the techniques described below.

**Optimal:** the solution that the CC can obtain by minimizing the trip time averaged over all EVs that ask for advice. This solution in general is not an NE, thus it may not be followed by rational drivers.

**Greedy:** the CC only disseminates information on the road travel time, and on the occupancy and the charging time at the stations. Based on this knowledge, each EV independently selects the charging station and the route that are deemed to minimize its own trip time. In this case, the CC just informs the EVs without providing any advice, and the EV decision is taken disregarding the presence of other vehicles looking for a charging station.

First, one may wonder whether the solution obtained



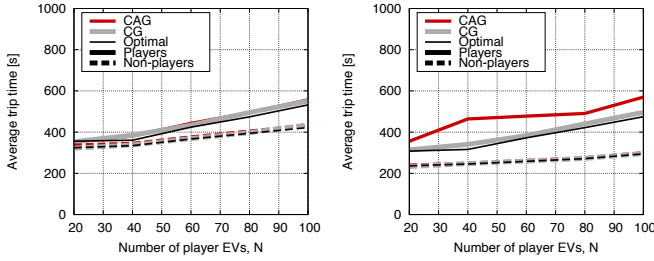


Fig. 4. Average per-EV trip time as a function of the number of players, when they represent 20% (left) and 60% (right) of all vehicles. CAG and congestion game (CG) are compared against the optimal.

through the CAG is as good as the one of the congestion game, or if the gain in complexity we have with the CAG takes a high toll in terms of system performance. To answer this question, in Fig. 4 we show the average vehicle trip time, for both player and non-player EVs, again as the number of players is 20% and 60% of the total number of vehicles. The performance corresponding to the solutions of the two games are also compared to that of the centralized optimal solution. The figure shows that the average trip times of player and non-player EVs have the same qualitative behavior, with the former clearly being higher than the latter since players stop at a charging station during their trip. As for the comparison among the CAG, the congestion game and the optimal, the difference in performance can be barely noticed when the players are 20% of the total number of EVs (left plot). When the percentage of players is large (right plot), the difference with respect to the optimal is limited in the case of the CAG, and it is again unnoticeable for the congestion game. This indicates that neglecting the contribution of player EVs to the travel time makes the CAG model less precise only when the players asking for advice in a given  $T$ -second period represent the majority of vehicles on the road topology.

Next, we investigate the benefit of our approach with respect to the greedy scheme. In spite of EVs receiving periodically updated information, Fig. 5 clearly shows that a greedy approach cannot cope with the other techniques in terms of performance, succumbing to a problem similar to the well-known route-flapping effect. Fig. 5 also depicts the performance of the CAG when the CC does not solve the game using the first NE that is reached, but the NE that minimizes the average per-player trip time among the first 10 it finds. In the plots, we label this curve by CAG-10. Interestingly, such a simple enhancement to the solution procedure makes the CAG approach as effective as the congestion game and the optimal, without impairing its scalability.

## V. CONCLUSIONS AND FUTURE WORK

Leveraging the use of ITS, we envisioned the availability of a navigation service that provides electric vehicles (EVs) that are low in battery with advice on the charging station to use and the route to take. We focused on how to determine such advice so that rational EV drivers find it convenient to themselves and they are willing to follow it.

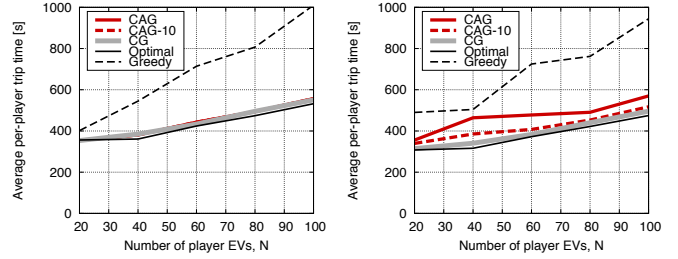


Fig. 5. Average per-player trip time vs. number of players, when they represent 20% (left) and 60% (right) of all vehicles. Comparison among CAG, congestion game, optimal, and greedy. CAG-10 refers to the CC taking as a solution the best among the first 10 NEs it finds.

After highlighting that traditional optimization approaches fail to achieve the above goal, we considered a realistic scenario and modeled the problem as a congestion game, for which at least one pure-strategy Nash equilibrium exists (i.e., a solution that all EVs find it satisfactory). Then, in order to lower the complexity, we introduced a game with congestion-averse utilities (CAG) that applies to a slightly simpler scenario but for which an NE can be found in polynomial time. Simulation results show that using CAGs, not only is viable, but the model solution also leads to a performance that is remarkably close to the optimum and much better than that attained with a greedy scheme.

Future work will consider other road topologies as well as vehicular traffic scenarios. It will also address the cases where not all EV drivers are rational and where the information collected through ITS and available at the CC may be partial or not fully accurate.

## ACKNOWLEDGMENTS

This work was supported by Regione Piemonte through the IoT\_ToI project.

## REFERENCES

- [1] TomTom, "How TomTom's HD TrafficTM and IQ RoutesTM data provides the very best routing," *White paper*, 2010.
- [2] F. Malandrino, C. Casetti, C.-F. Chiasserini, "A game-theoretic approach to EV driver assistance through ITS," *IEEE PIMRC*, 2012.
- [3] J. Ferreira, P. Pereira, P. Filipe, and J. Afonso, "Recommender system for drivers of electric vehicles," *International Conference on Electronics Computer Technology (ICET)*, 2011.
- [4] H. Qin, W. Zhang, "Charging scheduling with minimal waiting in a network of electric vehicles and charging stations," *ACM VANET*, 2011.
- [5] L. Zhao, P. Awater, A. Schafer, C. Breuer, A. Moser, "Scenario based evaluation on the impacts of electric vehicle on the municipal energy supply systems," *IEEE Power and Energy Society General Meeting*, 2011.
- [6] O. Sundstrom, C. Binding, "Flexible charging optimization for electric vehicles considering distribution grid constraints," *IEEE Trans. on Smart Grid*, vol. 3, no. 1, Mar. 2012.
- [7] R.W. Rosenthal, "A class of games possessing pure-strategy Nash equilibria," *International Journal on Game Theory*, vol. 2, no. 1, 1973.
- [8] A. Bye, M. Polukarov, N. R. Jennings, "Games with congestion-averse utilities," *Algorithmic Game Theory*, vol. 5814, no. 1, 2009.
- [9] K. Barry, "Battery-boosted quick-charger doesn't shock system," *WIRED*, Mar. 2012, [Accessed in July 2012].
- [10] F. Malandrino, C. Casetti, C.-F. Chiasserini, M. Reineri, "A game-theoretic approach to the selection of charging stations by EVs," *Tech. Rep.*, [https://dl.dropbox.com/u/801973/TechRep\\_EVpaper.pdf](https://dl.dropbox.com/u/801973/TechRep_EVpaper.pdf), 2012.
- [11] N. Nisam, T. Roughgarden, E. Tardos, V. V. Vazirani (Eds.), *Algorithmic game theory*, Cambridge University Press, 2007.